Domain 5: Diagnostic Assessment for Lessons 28–35

1. Kerri conducted an experiment with a number cube. The table shows the number of times that each number landed.

Times Landed

Number	Number of Tosses
1	8
2	10
3	7
4	6
5	9
6	8

For which number below did the experimental probability equal the theoretical probability?

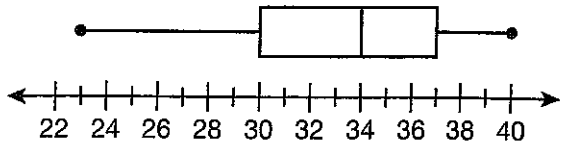
A. 1
B. 3
C. 4
D. 5

2. Jules wants to know which professional baseball player is most popular among the 300 students at his school. Which is the most representative and reasonable sample for him to survey?

A. collect data from a census of every student at his school
B. collect data from a sample of every student in his math class
C. collect data from a sample of every 20th student who arrives at school one day
D. collect data from a sample of every student on a school sports team

3. The box plot shows the heights of children who attend a library story time.

Children's Heights (in inches)

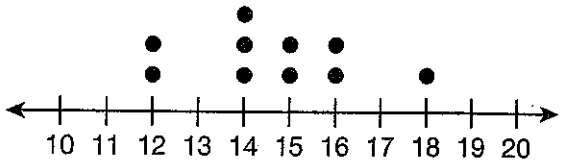


Which statement is **not** true about the data?

A. The range of the heights is 17 inches.
B. The interquartile range of the heights is 4 inches.
C. The median height of the children is 34 inches.
D. About 25% of the children are 30 inches or shorter.

4. The dot plot shows the number of miles Delia ran per week for 10 weeks. The data are a representative sample of the number of miles Delia runs throughout the year.

Number of Miles



Which best represents the mean number of miles Delia runs per week throughout the year?

A. 14
B. 14.5
C. 14.6
D. 15

5. The stem-and-leaf plots show the number of DVDs sold at two different stores during one month.

Store A

Stem	Leaves
2	0 6 7 8 8 9 9
3	2 2 3 5 7 8 9
4	1 3 5 5 5 8

Key: 2 | 0 = 20 DVDs

Store B

Stem	Leaves
2	0 1 3 5 6 6
3	1 3 3 4 8 9 9
4	2 5 5 6 7 8 9

Key: 2 | 0 = 20 DVDs

Which of the following statements about the median number of DVDs sold is true?

A. The median for Store A is the same as the median for Store B.

B. The median for Store A is 1 more than the median for Store B.

C. The median for Store A is 2 less than the median for Store B.

D. The median for Store A is 3 less than the median for Store B.

6. Mark will roll two number cubes. What is the probability that he will roll a sum of 6?

A. $\frac{1}{36}$

B. $\frac{5}{36}$

C. $\frac{1}{6}$

D. $\frac{5}{6}$

7. Gemma practiced the piano 5 days last week. The list below shows the number of minutes she practiced each day.

20, 25, 30, 20, 30

What is the mean absolute deviation in Gemma's practice times?

A. 4 minutes

B. 8 minutes

C. 12 minutes

D. 25 minutes

8. The table shows the results of two surveys Adam took to find out how many hours of sleep seventh-grade students at his school get on a typical school night.

Sample	Data
A	5, 6, 7, 8, 8, 8, 8, 8, 9, 9
B	7, 7, 7, 7, 8, 8, 8, 8, 10, 10

Based on the sample data, which of the following is most likely to be closest to the mean number of hours of sleep seventh-grade students at Adam's school get?

A. 7 hours

B. 7.75 hours

C. 8 hours

D. 8.25 hours

9. The heights, in feet, of four trees in a citrus grove are 8, 12, 10, and 14. What is the mean absolute deviation of the heights?

10. Middle school students in a school district volunteer in their community every month. The table shows the number of hours that students at one middle school volunteered for five months.

Community Hours

Month	Number of Hours
January	30
February	20
March	25
April	40
May	20

A. What is a reasonable prediction of the mean number of hours that students at the school will volunteer each month throughout the school year? Explain your thinking.

B. What is a reasonable prediction of the median number of hours that students at each middle school in the school district will volunteer each month throughout the school year? Explain your thinking.

Probability

Common Core State Standards:
7.SP.5, 7.SP.6, 7.SP.7.a, 7.SP.7.b

Getting the Idea

Probability measures the chance of an event happening based on the number of the **possible outcomes.** Probability can be expressed as a fraction or a decimal from 0 to 1. A probability close to 0 means an event is unlikely. A probability close to 1 means an event is very likely. A probability close to $\frac{1}{2}$ or 0.5 means an event is neither unlikely nor likely. You can also express a probability as a percent.

The **theoretical probability** of an event is the ratio of the number of ways the event can occur **(favorable outcome)** to the number of possible outcomes. The probability, P, of an event, A, is:

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}.$$

Example 1

Josh is going to choose a random card from 13 cards. The cards are numbered from 1 to 13. What is the probability that he will choose a card with a number less than 5? Determine if the event is likely, unlikely, or neither.

Strategy **Find the theoretical probability.**

Step 1 Count the number of favorable outcomes.

There are 4 cards (1, 2, 3, 4) with a number less than 5.

Step 2 Count the number of possible outcomes.

There are a total of 13 cards, each with the same chance of being drawn.

Step 3 Find the theoretical probability.

$$P(\text{card with a number less than 5}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{4}{13}$$

Step 4 Determine if the event is likely or unlikely.

$\frac{4}{13}$ is closer to 0 than it is to 1, and it is less than $\frac{1}{2}$. So, the event is unlikely.

Solution **The probability of choosing a card with a number less than 5 is $\frac{4}{13}$. The event is unlikely.**

You can use theoretical probability to make a prediction. Multiply the theoretical probability by the number of **trials**, or times the experiment is performed to predict the number of favorable outcomes.

Example 2

Peter will roll a number cube, labeled 1 through 6, a total of 90 times. What is a good prediction for the number of times that the number cube will land on 5?

Strategy **Find the number of possible outcomes and favorable outcomes.**

Step 1 Find the number of possible outcomes.

There are 6 possible outcomes for the number cube.

Step 2 Find the number of favorable outcomes.

There is one 5 on the number cube.

Step 3 Write the theoretical probability in simplest form.

$$P(\text{rolling a 5}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{1}{6}$$

Step 4 Multiply the probability by the number of trials.

$$\frac{1}{6} \times 90 = 90 \div 6 = 15$$

Solution **A good prediction is that Peter will roll a 5 about fifteen times.**

Experimental probability is the ratio of the total number of times the favorable outcome happens to the total number of trials, or times the experiment is performed. The experimental probability, P_e, of event A is:

$$P_e(A) = \frac{\text{number of favorable outcomes}}{\text{total number of trials}}.$$

Experimental probability is useful when you need to make predictions about an event. As the number of trials increases, the experimental probability gets closer to the theoretical probability.

Example 3

Minnie conducted an experiment with a spinner. The results are shown in the table.

Number	1	2	3	4	5	6
Times Landed	10	7	6	8	5	6

Based on the data, what is the probability that the spinner will land on 2 on the next spin?

Strategy **Find the experimental probability.**

Step 1 Find the number of trials.

$10 + 7 + 6 + 8 + 5 + 6 = 42$

Step 2 Find the number of favorable outcomes.

The spinner landed on 2 a total of 7 times.

Step 3 Write the experimental probability as a fraction in simplest form.

$\frac{7}{42} = \frac{1}{6}$

Solution **The experimental probability of the spinner landing on 2 on the next spin is $\frac{1}{6}$.**

Example 4

Gavin rolls a number cube, labeled 1 to 6, a total of 40 times. The number 4 is rolled 8 times. What is the experimental probability of rolling a 4? What is the theoretical probability? Describe the difference between the two.

Strategy **Use the formulas for experimental probability and theoretical probability.**

Step 1 Find the experimental probability.

The total number of trials is 40.

The number 4 is rolled 8 times, so the number of favorable outcomes is 8.

$$P_e(4) = \frac{\text{number of favorable outcomes}}{\text{total number of trials}} = \frac{8}{40} = \frac{1}{5}$$

Step 2 Find the theoretical probability.

The number of possible outcomes is 6.

There is only one 4 on a number cube, so the number of favorable outcomes is 1.

$$P_e(4) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{1}{6}$$

Step 3 Compare the experimental probability and the theoretical probability.

The experimental probability is $\frac{1}{5}$, and the theoretical probability is $\frac{1}{6}$.

The theoretical probability shows the outcome you would expect.

The experimental probability shows the outcome that actually occurred during the experiment.

Solution **The experimental probability of rolling a 4 is $\frac{1}{5}$. This is greater than the theoretical probability of $\frac{1}{6}$.**

Lesson Practice

Choose the correct answer.

Use the following event for questions 1 and 2.

Dan rolled a number cube 20 times.
The cube landed on the number 3 six times.

1. What is the experimental probability that Dan will roll a number 3 the next time he rolls the number cube?

   A. $\frac{3}{10}$

   B. $\frac{2}{5}$

   C. $\frac{3}{5}$

   D. $\frac{7}{10}$

2. Which best describes what would likely happen if Dan rolled the number cube another 80 times?

   A. There would be no change.

   B. The experimental probability would get farther from the theoretical probability.

   C. The experimental probability would exactly match the theoretical probability.

   D. The experimental probability would get closer to the theoretical probability.

3. Sonya wrote each letter of LEDBETTER on a separate index card and put the cards in a box. She picked one letter at random, put the card back, and then repeated the experiment. If she performed this experiment 90 times, which is the best prediction for the number of times that Sonya would pick a T?

   A. 10

   B. 20

   C. 25

   D. 30

4. A lightbulb manufacturer found that out of 200 lightbulbs, 15 were defective. How many lightbulbs should the manufacturer expect to be defective out of 2,400 lightbulbs?

   A. 40

   B. 80

   C. 120

   D. 180

5. Shia conducted an experiment with a spinner. The results are shown in the table below.

Number	1	2	3	4
Times Landed	8	4	5	3

Based on this data, how many times can Shia expect to spin a 1 in the next 20 spins?

A. 8

B. 7

C. 5

D. 3

6. There are 12 girls and 8 boys in Ms. Sander's class. Each day, she randomly asks one student to take attendance. In 180 school days, which is the best prediction for the number of times that the student will be a girl?

A. 72

B. 90

C. 99

D. 108

7. Blake tossed a coin 80 times. The coin landed on heads 60 times.

A. What is the experimental probability that Blake will toss heads? Show your work.

B. Is the experimental probability greater than or less than the theoretical probability? Explain your thinking.

Compound Events

Common Core State Standards:
7.SP.8.a, 7.SP.8.b, 7.SP.8.c

Getting the Idea

A **compound event** is a combination of two or more events. Compound events can be dependent or independent. Events are **independent** when the outcome of one event does not affect the outcome of a second event. When the outcome of one event affects the outcome of a second event, the events are **dependent.**

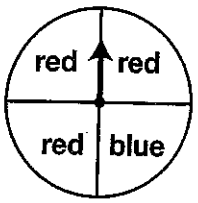
As with simple events, the probability of a compound event is the ratio of favorable outcomes to total outcomes in the sample space for which the compound events occur. You can use tables, organized lists, and tree diagrams to find the probability of compound events, or you can use the rules below.

To find the probability of two independent events, multiply the probability of the first event by the probability of the second event.

$P(\text{two independent events}) = P(\text{first event}) \times P(\text{second event})$

Example 1

Adriana tosses a number cube with faces numbered 1 through 6 and spins the spinner shown below at the same time.



What is the probability of tossing a number greater than 2 on the cube and spinning red on the spinner? Express the probability as a fraction, as a percent, and as a decimal.

Strategy **Find the probability of each event and multiply them together.**

Step 1 Decide if the events are dependent or independent.

The outcome on the number cube does not affect the outcome on the spinner.

The events are independent.

Step 2 Find the probability of the number cube landing on a number greater than 2.

A number cube has 6 possible outcomes.

Four outcomes (3, 4, 5, 6) are greater than 2.

$$P(>2) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{4}{6} = \frac{2}{3}$$

Step 3 Find the probability of spinning red on the spinner.

Three of the 4 sections are labeled "red."

$$P(\text{red}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{3}{4}$$

Step 4 Multiply the two probabilities.

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$$

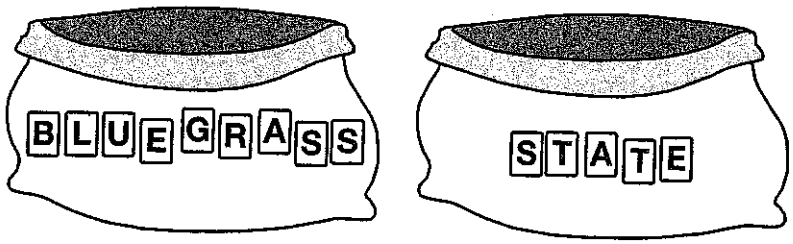
Step 5 Express the probability as a fraction, decimal, and percent.

$$\frac{1}{2} = 0.5 = 50\%$$

Solution **The probability of the cube landing on a number greater than 2 and the spinner landing on red is $\frac{1}{2}$, 0.5, or 50%.**

Example 2

Dion wrote the letters of the Kentucky state nickname on a set of same-sized cards and placed the cards into two bags as shown below.



He will choose one card from each bag without looking. What is the probability that he will choose the letter A from each bag?

Strategy **Find the probability of each event and multiply.**

Step 1 Decide if the events are dependent or independent.

The letter drawn from the first bag does not affect the letter drawn from the second bag.

The events are independent.

Step 2 Find the probability of choosing an A from the first bag.

One of the 9 letters is an A.

$P(A)$ for first bag $= \frac{1}{9}$

Step 3 Find the probability of choosing an A from the second bag.

One of the 5 letters is an A.

$P(A)$ for second bag $= \frac{1}{5}$

Step 4 Multiply the probabilities.

$\frac{1}{9} \times \frac{1}{5} = \frac{1}{45}$

Solution **The probability that Dion will choose an A from each bag is $\frac{1}{45}$.**

When you need to find the probability of a compound event, sometimes it is necessary to make a tree diagram, an organized list, or a table to find the number of possible outcomes.

You can also use the **fundamental counting principle** to find the number of possible outcomes. If event A can occur in m ways and event B can occur in n ways, then events A and B can occur in $m \times n$ ways.

Example 3

What is the probability of tossing a sum of 9 on two number cubes, each numbered 1 through 6?

Strategy **Find the number of possible outcomes. Then make an organized list.**

Step 1 Use the fundamental counting principle to find the number of possible outcomes.

There are 6 possible outcomes for each number cube.

$6 \times 6 = 36$, so there are 36 possible outcomes.

Step 2 Make an organized list of all of the ways to get a sum of 9 from the number cubes.

$3 + 6 \quad 4 + 5 \quad 5 + 4 \quad 6 + 3$

There are 4 ways to get a sum of 9.

Step 3 Write the probability in simplest form.

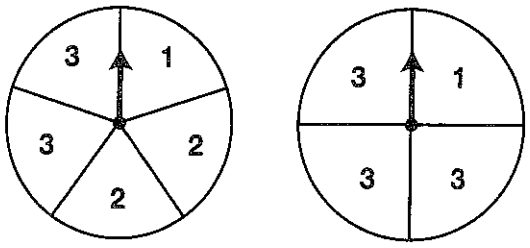
$\frac{4}{36} = \frac{1}{9}$

Solution **The probability of tossing a sum of 9 on two number cubes is $\frac{1}{9}$.**

Lesson Practice

Choose the correct answer.

Use the spinners below for questions 1 and 2.



1. What is the probability of spinning a 3 on both of the spinners?

   A. 0.25
   B. 0.3
   C. 0.35
   D. 0.4

2. What is the probability of spinning a sum of 4 when spinning both spinners at the same time?

   A. 20%
   B. 25%
   C. 30%
   D. 35%

3. Gabriel is doing a probability experiment. He is tossing a coin and spinning a spinner with 4 equal sections numbered from 1 through 4. How many possible outcomes are there?

   A. 2
   B. 4
   C. 6
   D. 8

4. There are 4 boys and 2 girls from the seventh grade and 3 boys and 5 girls from the eighth grade on the soccer team. Coach Hart will pick one captain from each grade. What is the probability that both captains will be girls?

   A. $\frac{5}{24}$
   B. $\frac{1}{4}$
   C. $\frac{3}{10}$
   D. $\frac{1}{2}$

5. Alexie tosses two dimes in the air. What is the probability that both dimes will land on heads?

A. $\frac{1}{2}$

B. $\frac{3}{8}$

C. $\frac{1}{4}$

D. $\frac{1}{6}$

6. Wendy is going to toss two number cubes with faces numbered 1 through 6 and a coin. How many possible outcomes are there for Wendy's experiment?

A. 14

B. 38

C. 72

D. 216

7. Patrick tosses a penny and a number cube, with faces numbered 1 through 6, at the same time.

A. Are the events dependent or independent? Explain your thinking.

B. What is the probability that the penny will land on heads and the number cube will land on a multiple of 3? Show your work.

Common Core State Standards:
7.SP.2, 7.SP.4

Make Predictions Using Data

Getting the Idea

You can use the data from a sample to make predictions about the population. It is important that the sample be representative of the population for the predictions to be reasonable.

Example 1

There are 60 students who take band classes at Mr. Tempo's school. Mr. Tempo surveyed 10 of those students to find out how long they practice their instruments each day. The survey was randomly distributed and anonymous. The results of the survey are shown below. The times are in minutes.

40, 25, 30, 40, 20, 15, 25, 30, 20, 25

Find the mean practice time for the sample. Predict the mean practice time of all the students who take band classes. Is the prediction reasonable?

Strategy **Use the mean from the sample data to predict the mean for the population.**

Step 1 Identify the sample and the population.

The students surveyed are the sample.

The population is all the students who take band classes.

Step 2 Find the mean for the sample data.

$40 + 25 + 30 + 40 + 20 + 15 + 25 + 30 + 20 + 25 = 270$

$270 \div 10 = 27$

The mean practice time is 27 minutes.

Step 3 Predict the mean for the population.

The mean practice time for the sample is 27 minutes.

The mean for the population should be about 27 minutes.

Step 4 Decide if the prediction is reasonable.

The sample was a random sample.

The size of the sample (10) is fairly large compared to the population (60).

The prediction is reasonable.

Solution **The mean practice time for the sample is 27 minutes. The sample mean provides a reasonable prediction of the population's mean practice time.**

Example 2

Mindy is the captain of the dance team at her school. She is running in a class election for class president. April surveyed the students on the dance team to see whom they planned to vote for in the election. The results of her survey are shown below.

Student	Number of votes
Mindy	18
Tobey	6
Roland	7

Based on the survey, predict who will win the class election. Is the prediction reasonable?

Strategy **Use the data to make a prediction. Evaluate the data to decide if the prediction is reasonable.**

Step 1 Use the data to make a prediction.

From the data in the table, the student with the greatest number of votes is Mindy.

Based on the data, Mindy should win the election.

Step 2 Evaluate the data to decide if the prediction is reasonable.

Mindy has quite a few more votes than either of the other students in the table.

However, Mindy is the captain of the dance team. All of the students who were surveyed are on the dance team. This suggests that they might be biased toward Mindy.

The prediction that Mindy will win does not seem reasonable.

Solution **Although the results of the survey suggest that Mindy will win, the survey is biased. The prediction that Mindy will win is not reasonable.**

Lesson Practice

Choose the correct answer.

1. The heights of five pepper plants, in centimeters, selected at random from a greenhouse with 50 pepper plants are shown below.

20, 24, 18, 23, 26

Which is a reasonable prediction of the mean height of all the pepper plants in the nursery?

A. 19 cm
B. 22 cm
C. 25 cm
D. 26 cm

2. A city is having a clean-up day for the 25 parks in the city. The list below shows the ages of volunteers participating in the clean-up project at one of the parks.

17, 18, 15, 16, 24, 20, 16

Which is a reasonable prediction of the mean age of all the participants in the clean-up project?

A. 16 years old
B. 17 years old
C. 18 years old
D. 19 years old

3. Ines read the following number of pages each day last week: 76, 123, 84, 110, 36, 20, and 90. Which is a reasonable prediction of the mean number of pages she reads each day throughout the year?

A. 75
B. 85
C. 95
D. 100

4. Vincent stood by the log flume at an amusement park and asked 120 people exiting the ride to name their favorite ride. The table below shows the results of his survey.

Favorite Ride

Ride	Number of People
Roller coaster	25
Ferris wheel	22
Log flume	58
Carousel	15

Which of the following is the most reasonable prediction based on the survey?

A. The results of the survey that show the log flume is the favorite ride are biased results because only log flume riders were surveyed.
B. The results of the survey that show the log flume is the favorite ride are reasonable results.
C. The results of the survey that show the log flume is the favorite ride are unreasonable because the number of people surveyed was so small.
D. The results of the survey that show the roller coaster is the favorite ride are unbiased results.

5. Carla runs for exercise several days each week. The number of miles she ran each week for the last 6 weeks is shown below.

10, 9, 8, 14, 9, 12

Which is a reasonable prediction of the mean number of miles Carla runs each week throughout the year?

A. 14

B. 13

C. 12

D. 10

6. Middle school students in a school district are collecting books to share with sister schools in Africa. The table shows the number of books that students at Lincoln Middle School collected for five months.

Books Collected

Month	Number of Books
August	60
September	50
October	30
November	40
December	70

A. What is a reasonable prediction of the mean number of books that students at Lincoln Middle School will collect each month throughout the school year? Explain your thinking.

B. What is a reasonable prediction of the mean number of books collected each month by the students at each middle school in the school district? Explain your thinking.